

## THE MODELLING OF A ROBOTIC ARM WITH FLEXIBLE LINKS

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**Abstract:** The purpose of the paper is to provide robot arm, which can be bent so that it can reach the same point by way of a great number of curvature combinations and thereby provide a very high accessibility, which means, that it can even pass obstacles of different kinds or bend itself around an object. Several methods from structural analysis have previously been used to model the articulated robotic arm, principally lever and beam structures, but these have frequently shown discrepancies when compared with experimental data. As an alternative, an arch representation for the robotic arm is considered here and allows the establishment of a criterion for the failure of the robotic arm that may be useful in determining absolute maximum loading conditions. It is proposed that the location of the thrust line in relation to the centre-line of the articulated arm is a useful predictor, and optimization techniques have been developed to find the best-fitting thrust line for the statically indeterminate structure.

### 1. INTRODUCTION

Medical robotics are known in a number of different embodiments and they usually consist of a machine, which without manual supervision or control can change the position of an object or a tool in a three dimensional space to a number of alternative points. The main portion of the medical robot is its robotic arm with its associated motion generating control system and program equipment, which can get a mini-computer for example. Advanced robots have a robot arm with up to six degrees of freedom, i.e. a possibility to move in six different planes, for example motion forwards, back-wards, upward, downwards, rotation to the left and rotation to the right. Conventional robot arms are built up from a number of elements and joints, which besides the tool and the load also must support the equipment for the motion and power generation for the separate elements. The purpose of the paper is to provide robot arm having a very broad working range and a maximum motion pattern, whereby is meant that it will reach almost all point inside a spherical working field.

A further purpose is to provide an arm with a very high rigidity in the the element plane of curvature and a high torsion resistance and which is cheaper to manufacture as compared to conventional robots. This has according been achieved by each element having single or double-curved segments or flat surfaces, and combinations of flat and/or curved surfaces. The curved contact surfaces of said segments each being located to contact a contact surface of the adjacent segment. The elements, being arranged to perform a rolling motion in relation to each other when actuated by said power-generating and/or power-transmitting actuating means. The articulated robotic arm is a three-dimensional mechanical structure that transmits loads and allows motion. The model system consists of two concentric tubes. We refer to these tubes as two mechanisms, inner and outer mechanism. Each tube can alternate between two modes: rigid mode or flexible mode. In rigid mode, the mechanism is just that – rigid. In flexible mode, the mechanism is highly flexible and thus either assumes the shape of its surroundings or can be reshaping in some other way.

Both inner and outer mechanisms consist of rigid cylindrical links connected by a type of spherical joint which can rotate  $\pm 15$  degrees range in both degrees of freedom. The links are maintained in contact by means of cable (wires); three cables for the outer mechanism and one cable for the inner mechanism. The three outer cables of the outer mechanism are 120 degrees apart, making it possible to steer in any direction, as well as selecting between rigid or flexible mode.

When the cables are pulled towards the back of the mechanism, the links are pulled towards each other increasing friction between the links eventually causing the outer mechanism to become rigid; when they are relaxed, the outer mechanism becomes limp. If

outer mechanism is coupled on flexible mode; it can manage external link of cinematic chain which will be moving forward. By alternate the two modes, flexible/rigid, of the two mechanisms, the inner and the outer one, the tip of the robotic arm of the device follows a curvature in 3 D space.

The robotic arm contains a multitude of adjacent elements (links) interconnected through cables (wires) arranged in a series. Each element is made with inverse segments (contrary, opposed) simply or double curved. The modular structure in which the links are attached one toward another is a benefit, because it allows a simple adjustment of the length, by positioning active sections of bending along the robot arm. This is important for the testing of various configurations in real time, as well as to assure an optimum performance of the charges applied.

An understanding of the articulated robotic arm is important in the study of its normal functional conditions, but current understanding is limited by the structural complexity of the medical robotic arm and the difficulty of in vivo experiments (with human subjects). Hence many mathematical models of the robotic arm have been developed, and these are principally classified as levers, simple beams, cantilever beams and arches.

Lever models typically describe the whole robotic arm as a rigid lever without consideration of curvature, and loading of the robotic arm is balanced by reaction forces at the sacrum. Simple beam models describe the whole robotic arm as a straight slender elastic beam, and Euler's theory of elastic buckling is used for analysis. Cantilever models describe the whole robotic arm as a cantilever beam rigidly built into the pelvis with applied forces and moments. Arch models describe either a part or all of the robotic arm as an arch. Stability under a variety of loading conditions can then be determined by compliance with the criterion that the thrust line should be completely located within the core of the arch articulated arm.

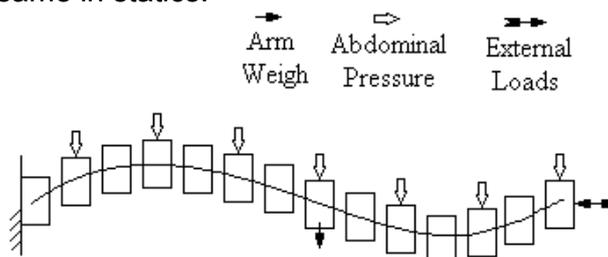
## 2. APPLIED LOADS

The loading system of the robotic arm can be simplified as arm weight, external loads, and abdominal pressure, as shown in figure. 1. The robotic arm structure may be approximated as a column consisting of independent block layers. Each structure is statically indeterminate as the number of unknowns is greater than the number of equations available for their solution.

The tension in the cables, the frictional forces between the layers (links) and the connection conditions between the cables and the base all are unknown. A loaded arch (figure 2) is statically indeterminate, as the reaction forces  $H_1$ ,  $H_2$ ,  $R_1$  and  $R_2$  cannot be found using force and moment equilibrium equations such as:

$$\sum f_x = 0 \quad \sum f_y = 0 \quad \sum M_A = 0 \quad (1)$$

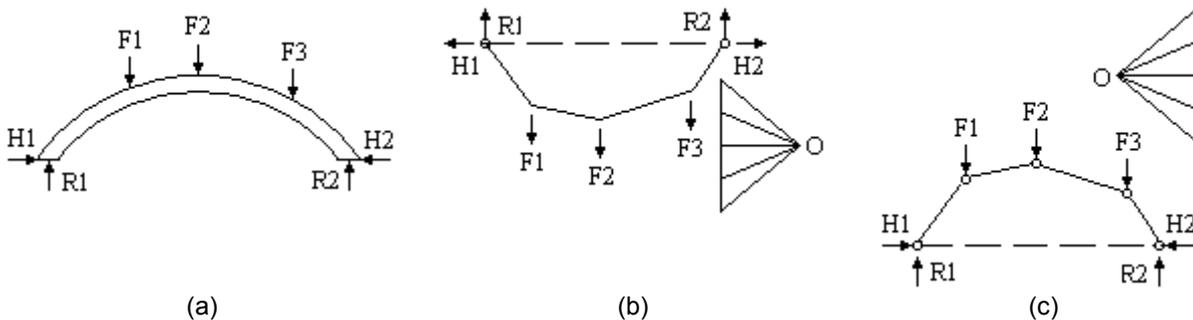
The forces polygon is an analysis tool that generates alternative solutions (figure 2b). The problems of the hanging cables (wires) and the arch which is modelled as rigid rods with joints (figure 2c) are the same in statics.



**Fig. 1 Loading system of the robotic arm**

The rods work in compression while the cables (wires) are in tension. Compressive force, thrust, is transmitted along a line called a thrust line. The section of the arch voussoirs which contain a thrust line and the force polygon determines the stability of the arch. If the horizontal component  $H$  ( $H_1 = H_2$  in this case) is known, then the forces polygon in figure 2c can be drawn from the force polygon.

The pole  $O$  could be placed anywhere, so an infinite number of thrust lines corresponding to different pole positions could be found. Loads applied in general directions are treated in a similar way to vertical loads. The position and shape of the force polygon again changes with the pole  $O$  of the force polygon.



**Fig. 2** *Sting and arch in equilibrium; (a) masonry arch, (b) string in equilibrium, (c) arch in equilibrium*

According to the middle-third rule, if the applied load (thrust line) stays within a core of the section, stresses across the whole section will be compressive and the arch is safe. For a rectangular section the core has a depth of one-third of the total depth. The existence of a satisfactory thrust line ensures that the arch cannot collapse, whereas thrust lines outside the arch cause collapse due to the formation of joints. The robotic arm can be modelled as an arch with the assumptions that:

- loads are transmitted by compressive forces along the spine;
- normal compressive forces are lower than the crushing strength of vertebrae and discs sliding failure cannot occur.

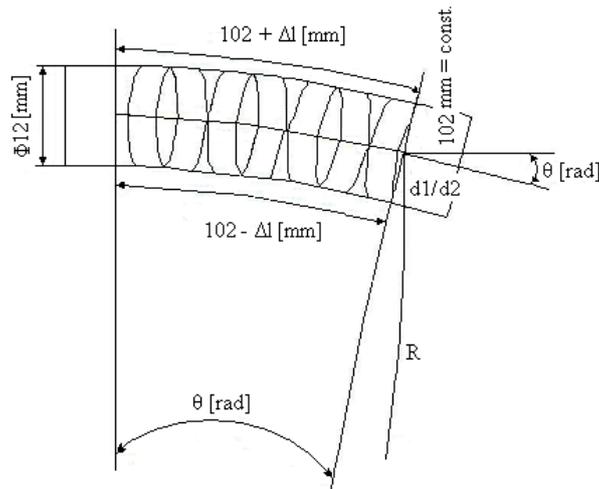
The robotic arm is similar to the masonry arch but the factor of safety may be different. Also, the centre-line of the core may not be located at the central line of the vertebrae of the arm. A hinge forms as a thrust line approaches the surface of the vertebrae, as shown in figure 2a.

### 3. MATHEMATICAL MODEL FOR FLEXIBLE LINKS

Simplified mathematical model of the flexible links is as follows (figure 3). If we assume that the length of the flexible links is 102 mm, pulling of the wire underneath the links will cause the links to bend with radius  $R$ . Each link having the radius of  $d$ , the outer and inner radii of the arc will have  $102 + \Delta l$  and  $102 - \Delta l$ , respectively, where  $\Delta l$  is the change in length due to bending. Assuming the same bead length, joint friction and no slipping at the joint, we may come up with following equations:

$$d \times \theta = \Delta l \quad (2)$$

$$(R - d) \times \theta = (25 - \Delta l) \quad (3)$$



**Fig. 3 Mathematical Model of Flexible Links**

Based on the above equations, we can assume a linear relationship between  $\theta$  and  $\Delta l$  and may be able to determine its validity through experiments. Therefore, we will continue development of the device focusing on a smaller diameter, increased stiffness and better curvature capabilities.

#### 4. STRUCTURE OPTIMIZATION

An infinite number of thrust lines for an arch exist as it is a statically indeterminate structure. Finding the best (closest to the centre-line of the arch) thrust line is an optimization problem. A criterion of failure for the articulated arm is then:

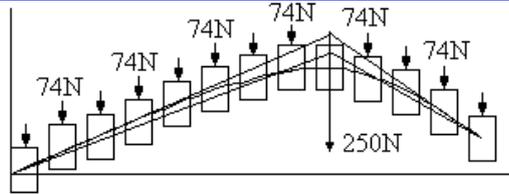
- the best-fitting thrust line among all thrust lines in a the articulated arm can be found but it is not located within the inside of the complete arch the articulated arm;
- then, the the articulated arm fails or disorders occur.

To find the best-fitting thrust line it is necessary to find the best position of the pole O of the force polygon. This is a 2D non-linear optimization problem, and four objective functions to achieve this are presented as follows:

$$\begin{aligned}
 f_1 &= \min[\max |d_i|]; & i &= 1, n \\
 f_2 &= \min[\sum d_i^2]; & i &= 1, n \\
 f_3 &= \min[\sum (w_i / d_i)]; & i &= 1, n \\
 f_4 &= \min[w_i / d_n + w_2 \max |d_i|]; & i &= 1, n - 1
 \end{aligned}
 \tag{4}$$

where:  $|d_i|$  - is the distance between the thrust line and the arm central line;  $w$  - is a weighting factor which forces the thrust line to be as close as possible to the central line;  $n$  - is the number of forces.

The best result of several locally optimized positions of the pole O in the force polygon from the above four objective functions is chosen as the final optimization result. The articulated arm weight, intra-abdominal pressure and additional load carried are treated as the external loads. The optimization calculation results are shown in figure 4.



**Fig. 4 Optimization of the force lines**

Comparison between the optimization result and the data result reveals that the optimized thrust line is closer to the arm centre-line.

To determine the torques required from the motors, as well as the materials used at the device, an extreme (worst-case) configuration was considered, a configuration where the actuators have to exert the worst torque. Such an extreme configuration occurs when the device is stretched out in a cantilevered position, the outer mechanism is on the flexible mode, and the inner mechanism supports its own weight as well as the weight of the outer mechanism, figure 5 .

Has generated a simplified model of the extreme configuration to estimate the needed axial tension applied to the actuation cable (wire). For this simplified model, we approximated the system parameters: the outer mechanism link weight - 3,5 g, the inner mechanism link weight - 2,5 g, the number of links in each mechanism - 17, the total weight of the robotic arm - 102 g, the outer diameter of the outer mechanism - 12 mm, the outer diameter of the inner mechanism - 6 mm, the total length of the robotic arm - 240 mm.

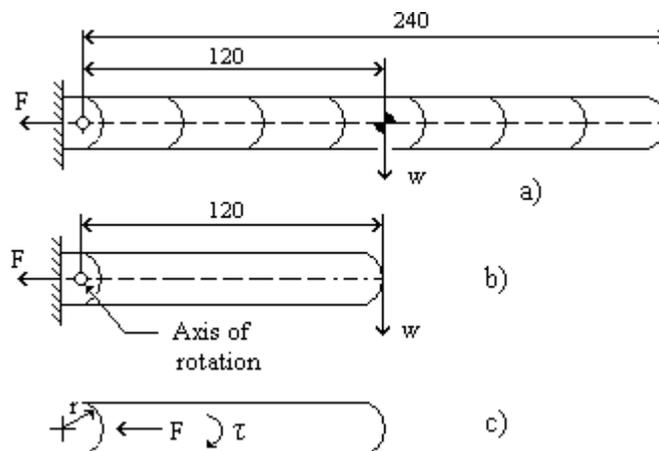
The weight of the arm console is thought to work at a point called center of mass. When the friction between surfaces of two links is small, the necessary drawing force for the mechanism to support its own weight is awfully big. So, the choice for the links material is a crucial factor.

The approximate relationship between the axial force ( $F$ ) and the torque  $\tau$  applied on a spherical surface with radius  $r$  and taking into account the coefficient of friction  $\mu$  is:

$$\tau = \mu \cdot F \cdot r$$

$$F = \frac{\tau}{\mu \cdot r} = \frac{90 \text{ Nmm}}{\mu \cdot 6 \text{ mm}} = \frac{15 \text{ N}}{\mu} \quad (5)$$

In order to estimate the torque required from the motor to pull the cable with the required force  $F$ , we use the relationship between the motor torque ( $\tau_{\text{motor}}$ ), the radius of the pulley ( $r_p$ ), and the pulling force on the cable ( $F$ ):



**Fig. 5 Schematic of extreme cantilever configuration; (a) Extrem configuration, (b) Simplified model; (c) Freebody diagram of extreme configuration**

$$\tau_{motor} = Fr_p \Leftrightarrow F = \frac{\tau_{motor}}{r_p} \quad (6)$$

As we observed in (2), in order to put a higher force per given motor torque, a small radius of the pulley is required ( $r_p \approx 3$  mm). In the current design, the outer mechanism, when is on the rigid mode, is necessary to withstand loads of about 5 - 10 N.

## 5 CONCLUSIONS

The robotic arm contains a multitude of adjacent elements (links) interconnected through cables (wires) arranged in a series. Each element is made with inverse segments (contrary, opposed) simply or double curved. The modular structure in which the links are attached one toward another is a benefit, because it allows a simple adjustment of the length, by positioning active sections of bending along the robot arm. This is important for the testing of various configurations in real time, as well as to assure an optimum performance of the charges applied.

A criterion of failure of the articulated arm is generated in a situation where, if the best-fitting thrust line among all of the thrust lines in a arm can be found but it is not located within the core of the complete arm, then the arm fails. The best-fitting thrust line which is as close to the reference line of the articulated arm as possible can be obtained using optimization techniques and is a better predictor compared with those reported in the literature.

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